Problem 1 integrate: $\int \frac{\sec(5x)}{\cot(5x)}\,dx$

Mathcad solution $\int \frac{\sec(5x)}{\cot(5x)}\,dx \rightarrow \frac{1}{5\cdot\cos(5\cdot x)}$

rearranging: $\int \frac{\sec(5x)}{\cot(5x)}\,dx = \int \frac{\sin(5\cdot x)}{\cos(5\cdot x)^2}\,dx = \int \sin(5x)\cdot\sec(5x)\,dx$

AND REMEMBERING

$\int u\,dv = uv - \int v\,du$

Integrating by parts where:

$u = \sin(5x)$ $du = 5\cdot\cos(5\cdot x)\cdot dx$

$dv = \sec(5\cdot x)^2\cdot dx$ $v = \frac{1}{5}\cdot\frac{\sin(5\cdot x)}{\cos(5\cdot x)}$ ***** see below**

$$\int \sin(5\cdot x)\cdot\sec(5\cdot x)^2\,dx = \sin(5\cdot x)\cdot\left(\frac{1}{5}\cdot\frac{\sin(5\cdot x)}{\cos(5\cdot x)}\right) - \int \frac{1}{5}\cdot\frac{\sin(5\cdot x)}{\cos(5\cdot x)}\cdot 5\cdot\cos(5\cdot x)\,dx = \frac{\sin(5\cdot x)}{5}\cdot\frac{\sin(5\cdot x)}{\cos(5\cdot x)} - \int \sin(5\cdot x)\,dx = \frac{\sin(5\cdot x)}{5}\cdot\frac{\sin(5\cdot x)}{\cos(5\cdot x)} + \frac{1}{5}\cdot\cos(5\cdot x)$$

$$= \frac{\sin(5x)}{5}\cdot\frac{\sin(5x)}{\cos(5x)} + \frac{1}{5}\cdot\cos(5\cdot x) = \frac{\sin(5x)^2}{5\cdot\cos(5x)} + \frac{\cos(5x)}{5} = \frac{\sin(5x)^2}{5\cdot\cos(5x)} + \frac{\cos(5x)^2}{5\cdot\cos(5x)} = \frac{\sin(5x)^2 + \cos(5x)^2}{5\cdot\cos(5x)} = \frac{1}{5\cdot\cos(5x)}$$

So ... $\int \frac{\sec(5x)}{\cot(5x)}\,dx = \frac{1}{5\cos(5x)}$

***** for fun, let's show:** $\int \sec(5x)^2\,dx = \frac{1}{5}\cdot\frac{\sin(5x)}{\cos(5x)}$

we can do this by differentiating sin(x)/cos(x) to obtain the integrand thus: $\frac{d}{dx}\left(\frac{1}{5}\cdot\frac{\sin(5x)}{\cos(5x)}\right) = \sec(5x)^2$

remembering: $\frac{d}{dx}\frac{u}{v} = \frac{1}{v}\cdot\left(\frac{d}{dx}u\right) - \frac{u}{v^2}\cdot\left(\frac{d}{dx}v\right)$

where

$u = \sin(5x) \qquad du = 5\cdot\cos(5\cdot x)$

$v = 5\cos(5x) \qquad dv = -25\cdot\sin(5\cdot x)$

$$\frac{d}{dx}\frac{\sin(5x)}{5\cos(5x)} = \frac{1}{5\cos(5x)}\cdot(5\cdot\cos(5\cdot x)) - \frac{\sin(5x)}{(5\cos(5x))^2}\cdot(-25\cdot\sin(5\cdot x)) = 1 + \frac{25\cdot\sin(5x)^2}{25\cdot\cos(5x)^2}$$

$$= \quad 1 + \frac{\sin(5x)^2}{\cos(5x)^2} = \frac{\cos(5x)^2}{\cos(5x)^2} + \frac{\sin(5x)^2}{\cos(5x)^2} = \frac{1}{\cos(5x)^2} = \sec(5x)$$

Problem 2

$$\int \frac{1+\sin(2t)}{\tan(2t)}\,dt$$

Mathcad solution

$$\int \frac{1+\sin(2t)}{\tan(2t)}\,dt \rightarrow \frac{1}{2}\cdot\ln(\sin(2\cdot t)) + \frac{1}{2}\cdot\sin(2\cdot t)$$

lets simplify the integrand first:

$$\frac{1+\sin(2t)}{\tan(2t)} = \frac{\cos(2t)}{\sin(2t)} + \frac{\sin(2t)}{\frac{\sin(2t)}{\cos(2t)}} = \cot(2t) + \cos(2t)$$

using integration rules we have for these two trig functions yields:

$$\int \frac{1+\sin(2t)}{\tan(2t)}\,dt = \int \cot(2\cdot t) + \cos(2\cdot t)\,dt = \frac{1}{2}\cdot\ln(\sin(2t)) + \frac{1}{2}\cdot\sin(2t)$$

So ...

$$\int \frac{1+\sin(2t)}{\tan(2t)}\,dt = \frac{1}{2}\cdot\ln(\sin(2t)) + \frac{1}{2}\cdot\sin(2t)$$

Problem 3 $\int \cos(2x)^5\,dx$

Mathcad solution: $\int \cos(2x)^5\,dx \rightarrow \frac{1}{10}\cdot\cos(2\cdot x)^4\cdot\sin(2\cdot x) + \frac{2}{15}\cdot\cos(2\cdot x)^2\cdot\sin(2\cdot x) + \frac{4}{15}\cdot\sin(2\cdot x)$

Using the rule: $\int \cos(a\cdot x)^n\,dx = \frac{1}{n\cdot a}\cdot\cos(ax)^{n-1}\cdot\sin(a\cdot x) + \frac{n-1}{n}\cdot\int \cos(a\cdot x)^{n-2}\,dx$ where: $a = 2$ $n = 5$

$\int \cos(a\cdot x)^n\,dx = \frac{1}{10}\cdot\cos(2x)^4\cdot\sin(2\cdot x) + \frac{4}{5}\cdot\int \cos(2\cdot x)^3\,dx$ **equation [3A]**

but using the rule: $\int \cos(a\cdot x)^3\,dx = \frac{1}{3\cdot a}\cdot\sin(a\cdot x)\cdot\left(\cos(a\cdot x)^2 + 2\right)$ where: $a = 2$

$\int \cos(2\cdot x)^3\,dx = \frac{1}{6}\cdot\sin(2\cdot x)\cdot\left(\cos(2\cdot x)^2 + 2\right) = \frac{4}{3}\cdot\sin(x)\cdot\cos(x)^5 - \frac{4}{3}\cdot\sin(x)\cdot\cos(x)^3 + \sin(x)\cdot\cos(x)$

substituting into equation **[3A]** $\int \cos(a\cdot x)^n\,dx = \frac{1}{10}\cdot\cos(2x)^4\cdot\sin(2\cdot x) + \frac{4}{5}\cdot\left(\frac{4}{3}\cdot\sin(x)\cdot\cos(x)^5 - \frac{4}{3}\cdot\sin(x)\cdot\cos(x)^3 + \sin(x)\cdot\cos(x)\right)$

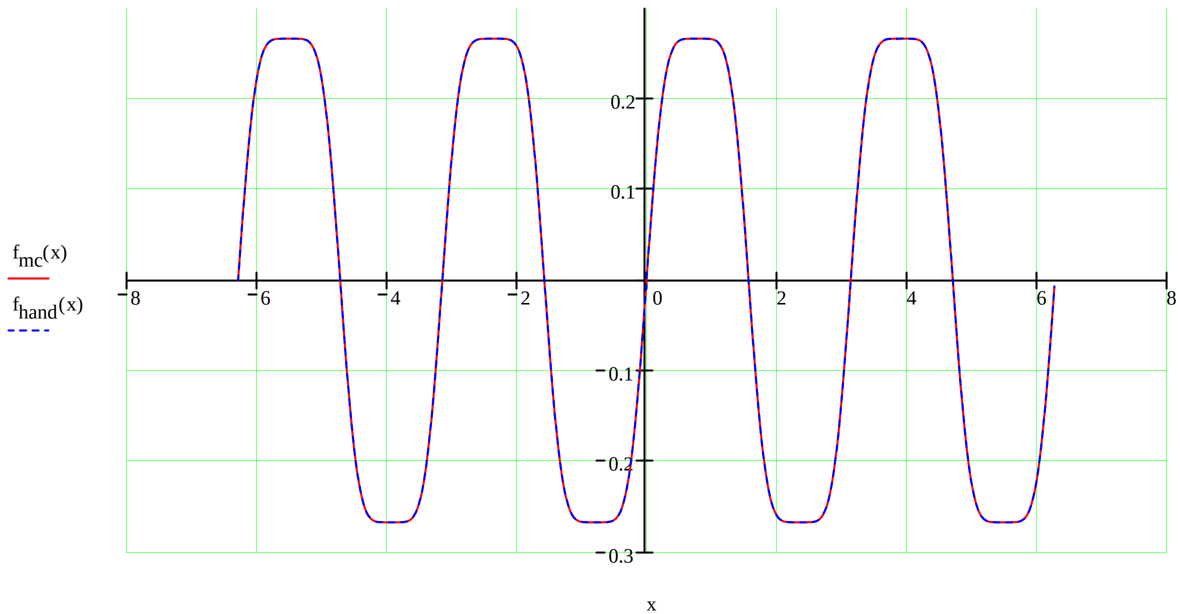
yields: $\int \cos(a\cdot x)^n\,dx = \frac{1}{10}\cdot\cos(2x)^4\cdot\sin(2\cdot x) + \frac{16}{15}\cdot\sin(x)\cdot\cos(x)^5 - \frac{16}{15}\cdot\sin(x)\cdot\cos(x)^3 + \frac{4}{5}\cdot\sin(x)\cdot\cos(x)$

Lets comparing the Mathcad solution with the hand solution over the domain: $x := -2\cdot\pi, (-2\cdot\pi + .01) .. 2\cdot\pi$

$$f_{mc}(x) := \frac{1}{10}\cdot\cos(2\cdot x)^4\cdot\sin(2\cdot x) + \frac{2}{15}\cdot\cos(2\cdot x)^2\cdot\sin(2\cdot x) + \frac{4}{15}\cdot\sin(2\cdot x)$$

$$f_{hand}(x) := \frac{1}{10}\cdot\cos(2x)^4\cdot\sin(2\cdot x) + \frac{16}{15}\cdot\sin(x)\cdot\cos(x)^5 - \frac{16}{15}\cdot\sin(x)\cdot\cos(x)^3 + \frac{4}{5}\cdot\sin(x)\cdot\cos(x)$$

Comparison of Mathcad and Hand Solution



The functions overlay and are identical.

Just for kicks, lets show the two functions are identical

given

$$\frac{1}{10}\cdot\cos(2\cdot x)^4\cdot\sin(2\cdot x)+\frac{2}{15}\cdot\cos(2\cdot x)^2\cdot\sin(2\cdot x)+\frac{4}{15}\cdot\sin(2\cdot x)=\frac{1}{10}\cdot\cos(2x)^4\cdot\sin(2\cdot x)+\frac{16}{15}\cdot\sin(x)\cdot\cos(x)^5-\frac{16}{15}\cdot\sin(x)\cdot\cos(x)^3+\frac{4}{5}\cdot\sin(x)\cdot\cos(x)$$

replacing sin(2x)

$$\frac{1}{10}\cdot\cos(2\cdot x)^4\cdot 2\cdot\sin(x)\cdot\cos(x)+\frac{2}{15}\cdot\cos(2\cdot x)^2\cdot(2\cdot\sin(x)\cdot\cos(x))+\frac{4}{15}\cdot(2\cdot\sin(x)\cdot\cos(x))=\frac{1}{10}\cdot\cos(2x)^4\cdot\sin(2\cdot x)+\frac{16}{15}\cdot\sin(x)\cdot\cos(x)^5-\frac{16}{15}\cdot\sin(x)\cdot\cos(x)^3+\frac{4}{5}\cdot\sin(x)\cdot\cos(x)$$

expanding

$$\frac{1}{5}\cdot\cos(2\cdot x)^4\cdot\sin(x)\cdot\cos(x)+\frac{4}{15}\cdot\cos(2\cdot x)^2\cdot\sin(x)\cdot\cos(x)+\frac{8}{15}\cdot\cos(x)\cdot\sin(x)=\frac{1}{5}\cdot\cos(2x)^4\cdot(\sin(x)\cdot\cos(x))+\frac{16}{15}\cdot\sin(x)\cdot\cos(x)^5-\frac{16}{15}\cdot\sin(x)\cdot\cos(x)^3+\frac{4}{5}\cdot\sin(x)\cdot\cos(x)$$

multiply through by 30

$$6\cdot\cos(2\cdot x)^4\cdot\sin(x)\cdot\cos(x)+8\cdot\cos(2\cdot x)^2\cdot\sin(x)\cdot\cos(x)+16\cdot\cos(x)\cdot\sin(x)=6\cdot\cos(2x)^4\cdot(\sin(x)\cdot\cos(x))+32\cdot\sin(x)\cdot\cos(x)^5-32\cdot\sin(x)\cdot\cos(x)^3+24\cdot\sin(x)\cdot\cos(x)$$

factoring

$$\sin(x)\cdot\cos(x)\cdot\left(6\cdot\cos(2\cdot x)^4+8\cdot\cos(2\cdot x)^2+16\right)=\sin(x)\cdot\cos(x)\cdot\left(6\cdot\cos(2x)^4+32\cdot\cos(x)^4-32\cdot\cos(x)^2+24\right)$$

dividing both sides by sin(x)cos(x)

$$6\cdot\cos(2\cdot x)^4+8\cdot\cos(2\cdot x)^2+16=6\cdot\cos(2x)^4+32\cdot\cos(x)^4-32\cdot\cos(x)^2+24$$

simplify

$$8\cdot\cos(2\cdot x)^2+16=32\cdot\cos(x)^4-32\cdot\cos(x)^2+24$$

simplify

$$\cos(2\cdot x)^2+2=4\cdot\cos(x)^4-4\cdot\cos(x)^2+3$$

simplify

$$\cos(2\cdot x)^2=4\cdot\cos(x)^4-4\cdot\cos(x)^2+1$$

trig identity

$$\left(2\cdot\cos(x)^2-1\right)^2=4\cdot\cos(x)^4-4\cdot\cos(x)^2+1$$

expanding

$$4\cdot\cos(x)^4-4\cdot\cos(x)^2+1=4\cdot\cos(x)^4-4\cdot\cos(x)^2+1$$

simplify

$0=0$ So, the functions are identical!

--------- some identities --------

$$\sin(2t)=2\sin(t)\,\cos(t)$$

$$\cos(2t)=\cos^2(t)-\sin^2(t)$$

$$=2\cos^2(t)-1$$

$$=1-2\sin^2(t)$$

$$\tan(2t)=\frac{2\tan(t)}{1-\tan^2(t)}$$

Problem 4

$$\int \tan(x)^5\,dx = \frac{-1}{2}\cdot\tan(x)^2 + \frac{1}{4}\cdot\tan(x)^4 + \frac{1}{2}\cdot\ln\left(1+\tan(x)^2\right)$$

Can you do this one???